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MAGNETIC INSULATION IN COAXIAL TRANSMISSION LINES WITH AN EXTERNAL MAGNETIC FIELD

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Abstract

A relativistic Brillouin flow equilibrium model is assumed for the plasma sheath and a cylindrical analytic solution including a uniform external magnetic field is obtained. A free parameter appearing in all previous magnetic insulation theories is shown to be accurately determined by maximizing the transmitted power flow. The theory is supported by particle-in-cell simulations.

The demands placed on high-voltage (multi-megavolt) technology have led to a great deal of interest in the phenomenon of magnetic insulation; i.e., the ability of magnetic fields to partially or completely insulate the space between two conductors held at a potential difference far exceeding the threshold for field emission. Magnetic insulation, as applied to the case of transmission lines, has been studied theoretically by several authors[1-2], but in all cases a parameter (e.g., the radial extent of the plasma sheath) has been left unspecified. The only exception has been early particle-in-cell simulations, with no external axial magnetic field, which indicate that magnetic insulation operates at the minimum possible current (in only partial agreement with the results here).[3] Thus, these theories offer no prediction for the actual operating current of a magnetically insulated transmission line. In addition, only in the planar approximation has the effect of an external magnetic field been included.[1]

A complete theory of magnetic insulation in a transmission line is necessary for the proper design of experiments requiring the generation of intense relativistic electron beams.[4,5] It is particularly important that the theory include the effects of an external magnetic field which is crucial to the operation of foilless diodes.[6] A foilless diode now appears to be a promising device for producing very intense, highly-collimated, annular relativistic electron beams.[7] Such a beam may be capable of producing a 10^{17}-10^{20} cm^{-3}, multi-kilovolt plasma with sufficient power density to implode a small cylindrical liner.[8]

In the theoretical analysis here, a complete theory of magnetic insulation in a cylindrical coaxial transmission line is presented by

using a relativistic Brillouin flow model (zero radial velocity) for the electron sheath. This is valid for the usual case where the voltage rise time is long compared with the cyclotron period of an electron in the combined external and self-magnetic fields.[9] In any event, the magnetic insulation solution appears to be insensitive to the particular flow model.[2] The theory presented here differs from previous analysis in two respects: (1) a uniform external magnetic field directed along the transmission line is included; (2) the free parameter in the previous theories is set here by maximizing the power flow while holding constant the voltage of the wave propagating down the transmission line. A very accurate cylindrical analytic solution is found.

This problem was also studied using the two-dimensional, time-dependent, fully relativistic and electromagnetic, particle-in-cell simulation code CCUBE.[10] Simulations were carried out at a variety of voltages, ratios of anode to cathode radii, and uniform external magnetic field strengths.

In the theoretical analysis, an equilibrium state with no azimuthal (θ) or axial (z) variation and no radial velocity (v_r) is assumed. Then the conservation of energy and canonical momenta immediately gives three integrals of the motion: $e\phi/mc^2 = \gamma-1$, $eA_z/mc = \gamma\beta_z$, and $eA_\theta/mc = \gamma\beta_\theta$, where the electrostatic (ϕ) and vector (A_z,A_θ) potentials and electron velocities $\beta_z \equiv v_z/c$ and $\beta_\theta \equiv v_\theta/c$ are assumed to vanish at the cathode radius $r = a$, and $\gamma^2 \equiv (1-\beta_z^2-\beta_\theta^2)^{-1}$. Here, e and m are the electron charge and mass, and c is the speed of light. The only remaining quantities to determine are β_z and β_θ. Amperes law,

Poisson's equation, and the above constants of motion can be combined to yield[11]

$$\beta_z'(\rho) = \beta_z'(1)/\rho\gamma^2 \tag{1a}$$

$$[\ln(\rho\gamma^2\beta_\theta')]' = \beta_\theta/\rho^2\beta_\theta' \tag{1b}$$

where the prime denotes $d/d\rho$, $\rho \equiv r/a$, and $\beta_z'(1) \equiv \beta_z'(\rho = 1)$ is a constant.

At present, we know of no exact analytic solution to Eqs. (1a) and (1b). The simulations, however, demonstrate that $\eta \equiv \beta_\theta/\beta_z$ is very nearly independent of ρ. Taking $\eta(\rho) = \eta(1)$, a constant, then Eq. (1a) gives

$$\gamma = \cosh\,[\alpha\beta_z'(1)\ln\rho] \tag{2a}$$

$$\beta_z = \alpha^{-1}\tanh\,[\alpha\beta_z'(1)\ln\rho] = \beta_\theta/\eta \tag{2b}$$

where $\alpha^2 \equiv 1 + \eta^2$. Equation (1b) in turn allows $\eta(\rho)$ to be estimated:

$$\eta(\rho) = \eta(1)\exp\{0.5[\alpha\beta_z'(1)]^{-2}\,\sinh^2\,[\alpha\beta_z'(1)\ln\rho]\}$$

or, for small ρ, $\eta(\rho) \cong \eta(1)\exp[(\ln\rho)^2/2]$. Thus, the approximation $\eta = \eta(1)$ is good if the plasma sheath remains relatively close to the cathode such as will occur for large external magnetic fields, large applied voltage (as discussed later), or sufficiently small ratio $(b-a)/a$ where b is the anode radius. The approximation $\eta = \eta(1)$ is

also applicable for sufficiently small external magnetic field, because then β_θ is negligibly small and Eq. (1b) becomes irrelevant.

Using the approximation $\eta = \eta(1)$, Eq. (2) applies in the region $1 \leqq \rho \leqq \rho_p$ where ρ_p designates the outer radius of the plasma sheath. Outside the plasma sheath ($\rho_p \leqq \rho \leqq b/a$), the electrostatic potential is given by $\gamma = [\gamma_0 \ln(\rho/\rho_p) - \gamma_p \ln(\rho a/b)]/\ln(b/a\rho_p)$, where $\gamma_p \equiv \gamma(\rho_p)$ and $\gamma_0 \equiv \gamma(\rho = b/a)$ is the anode potential. Continuity of the radial electric field at ρ_p, using this expression for γ and Eq. (2a), results in

$$\ln\rho_p = \ln(b/a)[1-\gamma_p(\gamma_0-\gamma_p)/(\gamma_p^2-1)G] \tag{3a}$$

$$G \equiv [\cosh^{-1}(\gamma_p) + (\gamma_0-\gamma_p)/(\gamma_p^2-1)^{\frac{1}{2}}]\ \gamma_p/(\gamma_p^2-1)^{\frac{1}{2}} \tag{3b}$$

Also, Eq. (2a) implies $\beta_z'(1) = \cosh^{-1}(\gamma_p)/\alpha\ln\rho_p$. Using $B_\theta = -dA_z/dr$, the total axial current (cathode boundary current plus electron sheath current) is $I_0 = I_A\beta_z'(1)\gamma_p$ with $I_A \equiv 2\pi mc/e\mu_0$.

So far the total anode voltage $V_0 \equiv (\gamma_0 - 1)mc^2/e$ has been treated as a parameter. Actually, in any experiment or time-dependent simulation V_0 is not a constant but is a function of the plasma sheath properties which vary with time. This is because the plasma alters the transmission line impedance and induces reflections. In practice a wave (e.g., right-going) of known fixed voltage $V_R \equiv (\gamma_R - 1)mc^2/e$ is launched from a driver of impedance Z_0 into the transmission line. The total steady-state voltage, including reflections, is then related to γ_R and the total current I_0 in the transmission line by

$$\gamma_0 = 1 + (\gamma_R - 1)(2 - I) \quad , \tag{4}$$

where $I \equiv I_0/I_R$ and $I_R \equiv V_R/Z_0$, so that determining the steady-state current I gives the appropriate γ_0. In the following discussion, Z_0 is chosen to be the vacuum impedance of the transmission line, $(\mu_0 c/2\pi)\ell n(b/a)$.

Using the above expression for $\beta_z'(1)$ along with Eqs. (3a) and (4), one finds

$$I = (\gamma_p^2-1)^{\frac{1}{2}}\, G/\alpha(\gamma_R-1) \quad . \tag{5}$$

The next step is to determine α. For $1 \leqq \rho \leqq \rho_p$ the azimuthal vector potential is $A_\theta = \gamma\beta_\theta mc/e$, while for $\rho_p \leqq \rho \leqq b/a$, it is $A_\theta = (\rho^2-\rho_1^2)k/\rho$. The constants ρ_1 and k are determined by matching A_θ at ρ_p and by setting $A_\theta(\rho = b/a) = (b^2-a^2)B_0/2b$ from conservation of axial magnetic flux. Note that the uniform external axial magnetic field B_0 has made its first appearance in the theory. Finally, matching the axial magnetic field at ρ_p from the above two expressions for A_θ requires that

$$\alpha = [1-\psi_0^2 U^2(b/a-a/b)^2/(\gamma_p^2-1)]^{-\frac{1}{2}} \tag{6a}$$

$$U \equiv \chi_p[1 + \chi_p^2+(1-\chi_p^2)G/\ln(b/a)]^{-1} \tag{6b}$$

where $\chi_p \equiv \rho_p a/b$ and $\psi_0 \equiv eB_0 a/mc$.

The coupled transcendental equations (3-6) form the basic analytic solution, with γ_p still a free parameter, and must in general be solved numerically. For $B_0 = 0$ this solution simplifies to that of Ref. 2.

The reason γ_p is as yet unspecified is that only the equilibrium equations have been solved. No indication of which equilibrium (out of a possible continuum) the full time-dependent equations would eventually lead to has yet been given. Whether this evolution through various other possible equilibrium states results from instabilities or statistical probabilities[12] is not discussed here, but the time-dependent simulations performed here demonstrate that the system does evolve to and oscillate about a unique steady-state. The oscillations could indicate that the system is trying to maximize or minimize some quantity. Furthermore, as B_0 increases I decreases toward unity until, for B_0 larger than some critical value, I remains clamped at unity. This is shown in Fig. 1a, which plots the operating impedance $Z/Z_0 \equiv V_0/I_0Z_0$ as a function of ψ_0 for $b/a = 1.853$ and two values of V_R (1.5 MeV and 6.4 MeV). Thus, the system appears to be maximizing some function of I, P(I), subject to some constraint on the range of I. As B_0 increases, the constraint is relaxed until I can reach closer and closer to unity where P attains its maximum value. The theory indeed shows that $I(\gamma_p)$ has a range of values that moves toward and eventually encompasses unity as B_0 increases from zero. A physical quantity having the desired form is $P(I) = I(2-I)$, the transmitted power flow down the transmission line in units of the launched power $P_R \equiv V_R^2/Z_0$. It is, therefore, proposed that magnetic insulation operates in the maximum power flow mode.[6]

By this it is not meant that $I = 1$, but rather that γ_p is chosen so that $I(\gamma_p)$ is as close to unity as the constraints allow. For sufficiently small B_0, $I(\gamma_p) > 1$ for all γ_p so that in this regime

power flow maximization is equivalent to current minimization (but remember V_R is held fixed, not V_0).

Using this argument to set γ_p in Eqs. (3-6), the theoretical Z/Z_0 is plotted as a function of ψ_0 in Fig. 1a. Also, plotted in Figs. 1b,c are the theoretical and simulation results for ρ_p and the ratio I_p/I_0 of plasma to total current as functions of ψ_0; Eq. (2) predicts $I_p/I_0 = (\gamma_p-1)/\gamma_p$, which extends the result of Reference (2) to include $B_0 \neq 0$. Due to temporal and spatial variations and limits on code resolution, the uncertainties of the simulation measurements are on the order of $\pm$ 5% for Z/Z_0 and ρ_p and $\pm$ 10% for I_p/I_0. In all cases, the theory and simulation are in close agreement.

Several interesting observations can be made from the theory: (1) Z/Z_0 is <u>independent</u> of Z_0 only for $B_0 = 0$ in which case $Z/Z_0 \cong 0.82[(\gamma - 1)/(\gamma + 1)]^{\frac{1}{2}}$; (2) ρ_p and I_p/I_0 at first <u>increase</u> as B_0 increases from zero but eventually fall abruptly as B_0 exceeds the critical value where $I = 1$; (3) the plasma sheath radius (maximized over all B_0) <u>decreases</u> (i.e., the effective insulation increases) as V_R increases; and (4) a plot of the total (radially integrated) energy per unit length along the transmission line as a function of γ_p does <u>not</u> reveal any consistent pattern (e.g., an extremum), valid for all V_R and B_0, that could be tested as an alternative principle for determining the operating γ_p. Observation (3) can be interpreted as due to the electrons needing less radial distance (for higher V_R) to acquire the necessary axial kinetic energy and hence axial current and B_θ required to insulate themselves. Observation (2) can be interpreted as due to the diversion of axial energy into azimuthal energy making it more difficult to generate an axial current; for large B_0, however, the axial magnetic field exceeds B_θ and dominates the insulation.

In conclusion, a complete theory (i.e., no free parameters) of magnetic insulation in a coaxial transmission line with an arbitrary axial external magnetic field has been presented. The analytic theory consists of the coupled transcendental equations (3)-(6) combined with the maximum power flow condition that $|I(\gamma_p) - 1|$ be minimized over all γ_p to determine the actual operating value of γ_p. Two crucial points were necessary in deriving this result: (1) the launched voltage V_R is the proper quantity to hold constant (not the total voltage V_0) when one calculates $I(\gamma_p)$ for different γ_p above; (2) including finite external magnetic field effects allows the solution to be studied as a function of an additional variable (B_0), and comparing this variation with simulation results leads to the maximum power flow condition.

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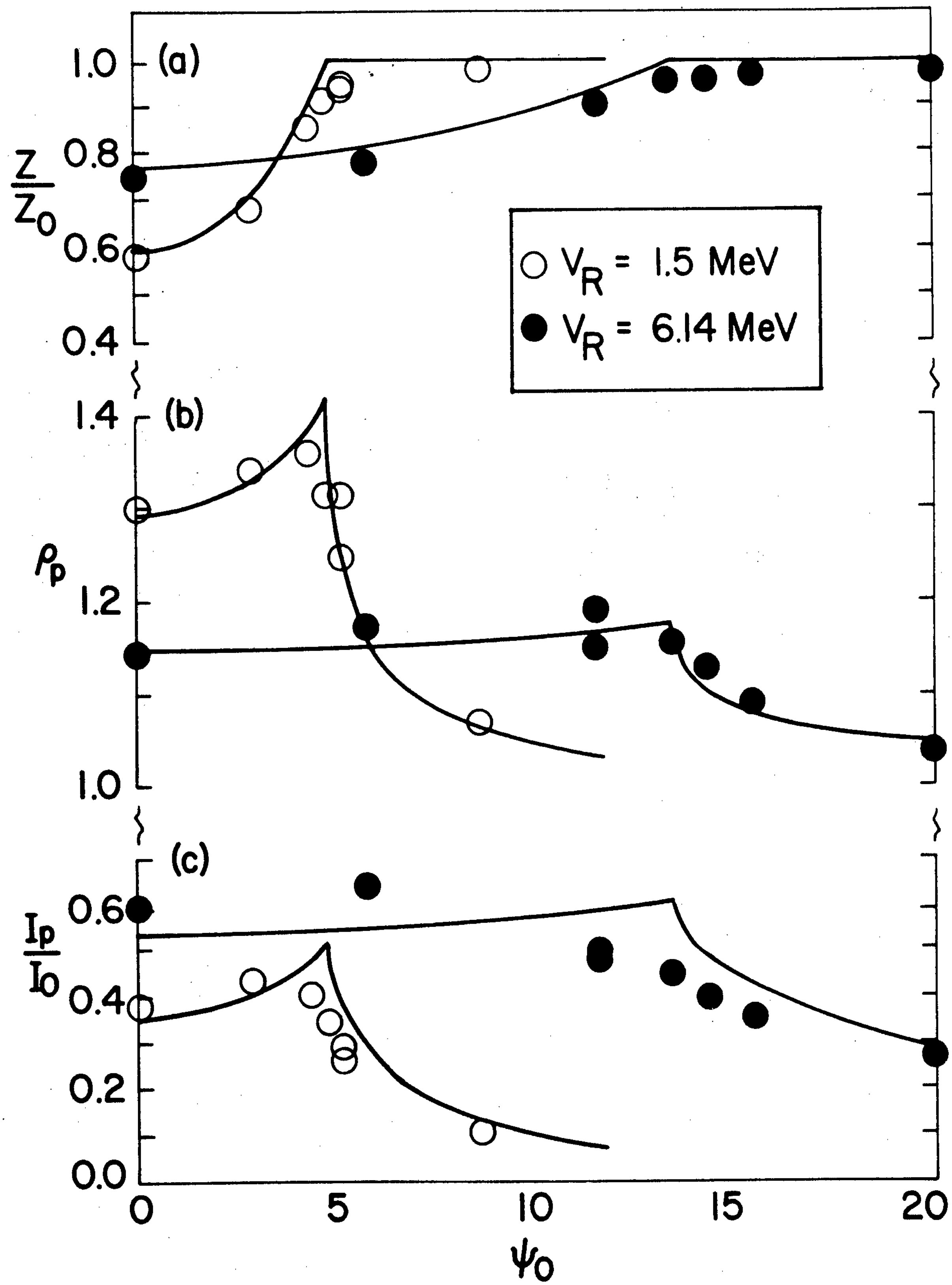
FIGURE CAPTION

Figure 1. Magnetic field variation of magnetic insulation impedance Z/Z_0, plasma sheath radius ρ_p, and ratio of plasma to total current I_p/I_0. Solid lines are laminar flow theory with power maximization. $\Psi_0 \equiv eB_0a/mc = 0.59\ B_0\ (kG)\ a\ (cm)$, $b/a = 1.853$.



(a)
(b)
(c)
Z/Z0
ρp
Ip/I0
1.0
0.8
0.6
0.4
1.4
1.2
1.0
0.6
0.4
0.2
0.0
0
5
10
15
20
ψ0
VR = 1.5 MeV
VR = 6.14 MeV